AQA

A-LEVEL
Mathematics

MM03 Mechanics 3
Mark scheme

6360
June 2016

Version 1.0 Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

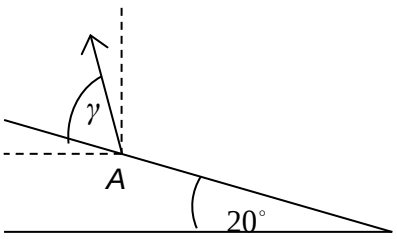
Q	Solution	Mark	Total	Comment
1 (a)	CLM:			
	$1.5(0)+0.03(0)=1.5(-v)+0.03(400)$	M1		M1: Correct terms, allow sign errors. OE, Condone the omission of the zero terms
			2	
	$v=8$ (ms^{-1}) OE	A1		A1: CAO
(b)				
	$I=1.5(8)-1.5(0)$	M1		M1: Seeing their *v* multiplied by 1.5 and condone the omission of the zero term
	$I=12$ (Ns)	A1	2	A1: CAO, must be positive.
	Total		**4**	

Q	Solution	Mark	Total	Comment
2 (a)	$[E]=\left[\frac{Gm_1m_2}{r}\right]$	M1		M1: Working with the 2nd term in the expression
	$=\text{MLT}^{-2}\text{L}^2\text{M}^{-2}\text{MML}^{-1}$	dM1		dM1: Correct unsimplified expression
	$=\text{ML}^2\text{T}^{-2}$	A1	3	A1: CAO
(b)	$\left[\frac{K^2}{2m_1r^2}\right]=\text{ML}^2\text{T}^{-2}$	M1		M1: Working with their answer to (a) and the first term of the expression
	$[K^2]=\text{ML}^2\text{ML}^2\text{T}^{-2}$	A1		A1: Correct dimensions for K^2
	$=\text{M}^2\text{L}^4\text{T}^{-2}$			
	$[K]=\text{ML}^2\text{T}^{-1}$	A1	3	A1:CAO
	Total		**6**	

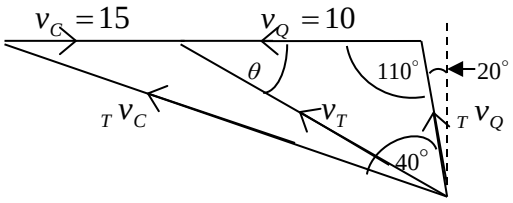
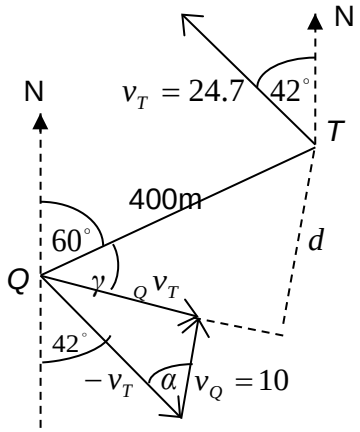
Q	Solution	Mark	Total	Comment
3(a)	$x = 14\cos 30^\circ t$	B1		B1: Correct horizontal eqn.
	$y = 14\sin 30^\circ t - \frac{1}{2}gt^2$	B1		B1: Correct vertical eqn.
	$t = \frac{x}{14\cos 30^\circ}$	M1		M1: Making t the subject of x
	$y = 14\sin 30^\circ \times \frac{x}{14\cos 30^\circ} - \frac{1}{2}(9.8)\left(\frac{x}{14\cos 30^\circ}\right)^2$	dM1		dM1:Elimination of t from their y
	$y = x\tan 30^\circ - \frac{x^2}{40\cos^2 30^\circ}$			
	$y = \frac{x\sqrt{3}}{3} - \frac{x^2}{30}$	A1	5	A1: CSO , AG must see the line above final answer, OE
(b)	$y = x - 6$	M1		M1: For seeing $y=x\pm 6$
	$x - 6 = \frac{x\sqrt{3}}{3} - \frac{x^2}{30}$	dM1		dM1: Substituting $x\pm 6$ into the given eqn.
	$x^2 + \left(30 - 10\sqrt{3}\right)x - 180 = 0$	A1		A1: Correct simplified quadratic
	$x = \frac{-\left(30 - 10\sqrt{3}\right) \pm \sqrt{\left(30 - 10\sqrt{3}\right)^2 - 4\times 1(-180)}}{2\times 1}$	dM1		dM1: Solving quadratic
	$x = 8.50$ $\left(x = -21.2 \text{ or exact equivalent not needed}\right)$	A1		A1: CAO, accept 8.5 or AWRT 8.50
	$PQ = \frac{8.499 - 6}{\cos 45^\circ}$	dM1		dM1: Correct exp. for PQ. FT their +ve x-value
	$PQ = 3.53$ or $\frac{5\sqrt{2}}{2}$ (m)	A1	7	A1: CAO, AWRT 3.53 or exact value, allow 3.54
	Total		**12**	

Alternative (b)	Solution	Mark	Total	Comment
	Let $PQ = d$.			
	$x = 6 + d\cos 45^\circ$	M1		M1:Expression for x in terms of d.
	$y = d\sin 45^\circ$	M1		M1:Expression for y in terms of d.
		A1		A1: Both correct
	$d\sin 45^\circ = (6 + d\cos 45^\circ)\tan 30^\circ - \dfrac{(6 + d\cos 45^\circ)^2}{30}$	dM1		dM1: Substituting into given expression
	$d^2\cos^2 45^\circ + (30\sin 45^\circ - 30\cos 45^\circ\tan 30^\circ + 12\cos 45^\circ)d - (180\tan 30^\circ - 36) = 0$	A1		A1: Correct simplified quadratic
	$d = \dfrac{-(30\sin 45^\circ - 30\cos 45^\circ\tan 30^\circ + 12\cos 45^\circ) \pm \sqrt{(30\sin 45^\circ - 30\cos 45^\circ\tan 30^\circ + 12\cos 45^\circ)^2 - 4(\cos^2 45^\circ)(36 - 180\tan 30^\circ)}}{2\cos^2 45^\circ}$	dM1		dM1: Solution of their quadratic eqn.
	$d = 3.53$ m (Allow 3.54 m)	A1		A1: CAO, AWRT 3.53 or exact value, allow 3.54

Q	Solution	Mark	Total	Comment
4 (a)(i)	$8mu + 4mu = mv_A + 4mv_B$ OE	M1 A1		M1: Four non-zero momentum terms, A1: Correct eqn.
	$7eu = v_B - v_A$ OE	M1 A1		M1: Eqn using e. Allow sign errors. A1: Correct eqn.
	$v_B = \frac{u}{5}(12 + 7e)$ OE	A1		A1: Correct vel of B
	$v_A = \frac{4u}{5}(3 - 7e)$ OE	A1	6	A1: Correct vel of A
(ii)	$\frac{4u}{5}(3 - 7e) < 0$	M1		M1: Their vel of A<0
	$e > \frac{3}{7}$	A1	2	A1: CAO, accept AWRT 0.429
(b)	$\frac{u}{5}(12 + 7e) \times \frac{2}{3} > -\frac{4u}{5}(3 - 7e)$	B1 M1		B1: Correct rebound speed of *B* M1: Correct inequality
	$e < \frac{6}{7}$	A1	3	A1: CAO, AWRT 0.857
(c)	$4m \times \frac{u}{5}\left(12 + 7 \times \frac{4}{7}\right) + 4m \times \frac{u}{5}\left(12 + 7 \times \frac{4}{7}\right) \times \frac{2}{3} =$	M1 A1		M1: Two correct momentum terms, allow sign errors. A1: Correct expression for impulse
	$\frac{64}{3}mu$ or $21.\dot{3}mu$	A1	3	A1: CAO (Must be positive)
	Total		**14**	

Q	Solution	Mark	Total	Comment
5 (a)(i)	$^{-}1 = 22\sin 50^\circ t - \frac{1}{2} g\cos 20^\circ t^2$	M1		M1: Perpendicular eqn with correct terms
		A1		A1: Correct equation
	$\frac{1}{2} g\cos 20^\circ t^2 - 22\sin 50^\circ t - 1 = 0$			
	$t = \dfrac{22\sin 50^\circ \pm \sqrt{\left(-22\sin 50^\circ\right)^2 - 4\left(\frac{1}{2} g\cos 20^\circ\right)(-1)}}{2\left(\frac{1}{2} g\cos 20^\circ\right)}$	dM1		dM1:Solution of their 3-term quadratic eqn.
			4	
(ii)	$t = 3.7185....$ or 3.719	A1		A1: CAO, AWRT 3.72
	$\dot{x} = 22\cos 50^\circ - 9.8\sin 20^\circ (3.7185....)$	M1		M1: Parallel component of vel. with their time
	$\dot{x} = 1.678$ ms^{-1}	A1		A1: Correct component, accept AWRT 1.68
			4	
	$\dot{y} = 22\sin 50^\circ - 9.8\cos 20^\circ (3.7185....)$	M1		M1: Perpendicular component of vel. with their time
	$\dot{y} = {}^{-}17.39$ ms^{-1}	A1		A1: Correct component, accept AWRT -17.4
(b)				
	$\gamma < 90^\circ - 20^\circ$	B1		B1: Seeing 90-20 or 70
	$\frac{17.39e}{1.678} < \tan(90^\circ - 20^\circ)$	B1F		B1F: Multiplying their vertical component by e
		M1		M1: Correct inequality
	$e < 0.265$	A1	4	A1: CAO, accept AWRT 0.265
	Total		**12**	

Q	Solution	Mark	Total	Comment
6 (a)	Along the line of centres:			
	CLM: $u\cos 60^\circ = v_A + v_B$ OE	M1		M1: Four non-zero momentum terms,
		A1		A1: Correct eqn.
	Restitution: $eu\cos 60^\circ = v_B - v_A$	M1		M1: Eqn using e. Allow sign errors.
		A1		A1: Correct eqn.
	$2v_B = (1+e)u\cos 60^\circ$ OE			
	$v_B = \frac{1}{4}u(1+e)$	A1		A1: Correct vel of B (AG) from correct working
	$v_A = \frac{1}{4}u(1-e)$ OE	A1		A1: Correct vel of A
	Perpendicular to line of centres: $v'_A = u\cos 30^\circ$ OE	B1	7	B1: Correct perpend. comp.
(b)				
	$v_A\cos 60^\circ + u\cos^2 30^\circ$ ← (A) ---------- Wall	M1		M1: Components of *A* parallel & perp to the wall
	$u\cos 30^\circ\cos 60^\circ - v_A\cos 30^\circ$ ↓	A1		A1: Both correct
	$v_B\cos 60^\circ$ ← (B) ---------- Wall	M1		M1: Components of *B* parallel & perp to the wall
	$e'v_B\sin 60^\circ$ ↓	A1		A1: Both correct AG above line oe needed
	$\frac{u\cos 30^\circ\cos 60^\circ - v_A\cos 30^\circ}{v_A\cos 60^\circ + u\cos^2 30^\circ} = \frac{e'v_B\sin 60^\circ}{v_B\cos 60^\circ}$	dM1		dM1: Equal ratios used to create equation
	$\frac{u\times\frac{\sqrt{3}}{2}\times\frac{1}{2} - \frac{1}{4}u(1-e)\times\frac{\sqrt{3}}{2}}{\frac{1}{4}u(1-e)\times\frac{1}{2} + u\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{e'\times\frac{\sqrt{3}}{2}}{\frac{1}{2}}$	A1		A1: Correct equation
	$e' = \frac{2u-u+eu}{u-eu+6u}$			
	$e' = \frac{1+e}{7-e}$	A1		A1:CSO (AG)
			7	
	Total		**14**	

Q	Solution	Mark	Total	Comment
7 (a)		B1		B1:For combined velocity triangles (PI)
	$\dfrac{25}{\sin 40^\circ} = \dfrac{{}_T v_Q}{\sin 30^\circ}$	M1		M1: Sine rule to find Vel of *T* rel to *Q*
	${}_T v_Q = \dfrac{25\sin 30^\circ}{\sin 40^\circ} \quad (= 19.4465)$	A1		A1: Correct expression or value
	$v_T = \sqrt{10^2 + \left(\dfrac{25\sin 30^\circ}{\sin 40^\circ}\right)^2 - 2\times 10 \times \dfrac{25\sin 30^\circ}{\sin 40^\circ} \times \cos 110^\circ}$	M1		M1: Cosine rule to find vel of *T*
	$v_T = 24.7(222662)$ ms^{-1}	A1		A1: Correct expression or value
	$\dfrac{\sin\theta}{\left(\dfrac{25\sin 30^\circ}{\sin 40^\circ}\right)} = \dfrac{\sin 110^\circ}{24.7222662}$	M1		M1: Sine rule to find θ
	$\theta = 47.6601^\circ$	A1		A1: Correct θ
	Bearing: 318°	A1	8	A1: Correct bearing (AG)
(b)		B1		B1:Right-angled velocity triangle
	$\cos\alpha = \dfrac{10}{24.7}$	M1		M1: Using trig to find α
		A1		A1: Correct equation to find α
	$\alpha = 66(.11775...)^\circ$	A1		A1: Correct α
	Motor cyclist's bearing: 024°	A1	5	A1: Correct bearing
	Total		**13**	

Q	Solution	Mark	Total	Comment
7(a) Alternative 1		B1		B1: For combined velocity triangles (PI)
	If v_T makes angle β with the north, then			
	$\dfrac{v_T}{\sin 30^\circ} = \dfrac{15}{\sin(60^\circ - \beta)}$	M1		M1: Sine rule to link v_T and β using 15, 30, 60.
	$\dfrac{v_T}{\sin 110^\circ} = \dfrac{10}{\sin(\beta - 20^\circ)}$	M1		M1: Sine rule to link v_T and β using 10, 110 , 20
	$15\sin 30^\circ \sin(\beta - 20^\circ) = 10\sin 110^\circ \sin(60^\circ - \beta)$	dM1		dM1: Correct eqn in β
	$15\sin 30^\circ(\sin\beta\cos 20^\circ - \cos\beta\sin 20^\circ) =$ $10\sin 110^\circ(\sin 60^\circ\cos\beta - \cos 60^\circ\sin\beta)$			
	$\tan\beta = \dfrac{10\sin 110^\circ \sin 60^\circ + 15\sin 30^\circ \sin 20^\circ}{15\sin 30^\circ \cos 20^\circ + 10\sin 110^\circ \cos 60^\circ}$	dM1	8	dM1: Finding tanβ
	$\beta = 42.3^\circ$	A1		A1: Correct β □
	Bearing : 318°	A1		A1: Correct bearing (AG)
	$\dfrac{v_T}{\sin 30^\circ} = \dfrac{15}{\sin(60^\circ - 42.3)}$ $v_T = 24.7$ ms^{-1}	A1		A1: Correct vel of *T*
Alternative 2				
	$v_T = \begin{pmatrix} a \\ b \end{pmatrix}$			
	${}_T v_Q = \begin{pmatrix} a+10 \\ b \end{pmatrix}$	B1		B1: Correct vector
	${}_T v_C = \begin{pmatrix} a-15 \\ b \end{pmatrix}$	B1		B1: Correct vector
	$\dfrac{a+10}{b} = -\tan 20^\circ$ OE	M1		M1: Correct ratio for tan20
	$\dfrac{a-15}{b} = -\tan 60^\circ$ OE	M1	8	M1: Correct ratio for tan60
	$a = -16.65\ldots \quad b = 18.27\ldots$	A1		A1: Both *a* and *b* correct
	Bearing : $270^\circ + \tan^{-1}\left(\dfrac{18.27}{16.65}\right) =$			
	318°	A1		A1: Correct expression for

	$v_T = \sqrt{(16.65...)^2 + (18.27...)^2}$ $v_T = 24.7 \text{ ms}^{-1}$	A1 A1		bearing A1: CAO (AG) A1: Correct vel of T